EUROPEAN CENTRAL BANK

BANKING SUPERVISION

Systemic Risk in Finance

Public Lecture Series

Lecture 2
Network construction and risk measures

Grzegorz Hałaj

Stress Test Experts Division, ECB & AI Lab SGH

Agenda

1. Partial information: Financial network (re)construction methods
2. Systemic risk measures: simple (mathematical) tools to capture systemic risk

Network (re)construction

Construction or reconstruction of financial networks?

- In Lecture 1, we saw how to build (construct a network)
  - Assuming that data are available (e.g., exposures between entities in the network like banks, investment funds, non-financial firms,…)
- But what if we have only limited data?
  - Can we **reconstruct a network**?
  - What information to rely on?...
  - …and if we have this information what methods to use to get a “good” model of the interconnectedness to measure systemic risk?

Reconstruction of financial networks - idea

- Banks, typically, report how much, in aggregate they borrow from other banks, how much they place at other banks
- Use this information to reconstruct how the network may look like
- Many configurations possible => we need assumptions

		B1	B2	B3
		50	100	50
B1	50	?	?	?
B2	50	?	?	?
B3	100	?	?	?

Any other way?

Find bilateral exposures that satisfy the marginals

		B1	B2	B3
		50	100	50
B1	50	0	50	0
B2	50	0	0	50
B3	100	50	50	0

How to approach this problem – first stab

Network represented by adjacency matrix: $A := \begin{bmatrix} x_{11} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots \\ x_{N1} & \cdots & x_{NN} \end{bmatrix}, x_{ij} \in \{0,1\}$

Or weighted matrix $A^w := \begin{bmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{bmatrix}, a_{ij} \geq 0$

Problem: **we know that $\sum_{i=1}^{N} a_{ij} = c_j$ and $\sum_{j=1}^{N} a_{ij} = r_i$, find "a" matrix A^w**

Naïve solution (always computable):

Denote $c^{(T)} := \sum_{i=1}^{N} c_i$ and $\boldsymbol{a_{ij}^{(n)} = \frac{r_i * c_j}{c^{(T)}}}$

How to approach this problem – example

	8	8	13
10	?	?	?
12	?	?	?
7	?	?	?

→

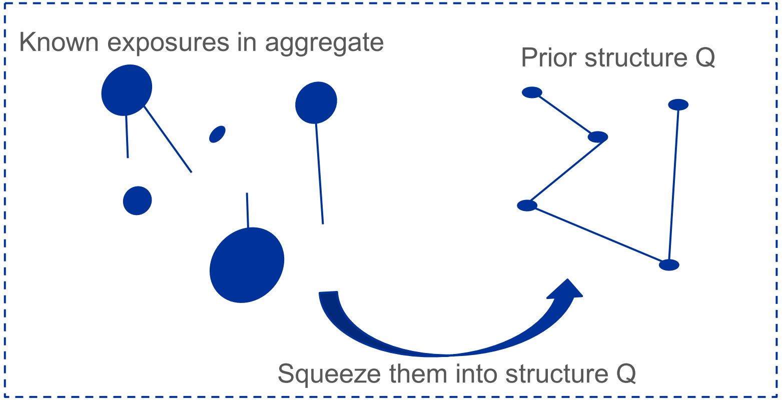
	8	8	13
10	2.76	2.76	4.48
12	3.31	3.31	5.38
7	1.93	1.93	3.14

e.g., 2.76 = 8*10/(10+12+7)

What if we some other priors about which links are more likely to be in the network?

Information theory and Kullback-Leibler divergence: of matrices P:= $[p_{ij}] = A^w/c^{(T)}$ from a prior likelihood Q:

$$D(P|Q) = \sum_{\substack{0\leq i\leq N \\ 0\leq j\leq N}} p_{ij}\log(\frac{p_{ij}}{q_{ij}})$$



Maximum entropy structure – important special case for uninformative prior (uniform Q)

Algorithm to compute a network based on a prior distribution

Maximum entropy structure, but not only, as this works for a general Q*

Algorithm 1 (RAS).

(**Step 1**) *Set the initial conditions* $k = 0$ *and* $\mathbf{Q}^{(0)} = \mathbf{Q}$.

(**Step 2**) *Increase the iteration index* $k \leftarrow k + 1$.

(**Step 3**) *For the* i^{th} *row,* $1 \le i \le m$, *find* $\alpha_i = p_{r_i} / \sum_{j=1}^{n} q_{ij}^{(k-1)}$ *and rescale each element of this row*

$$q_{ij}^{(k-1)} \leftarrow \alpha_i q_{ij}^{(k-1)}, 1 \le j \le n.$$

(**Step 4**) *For the* j^{th} *column,* $1 \le j \le n$, *find* $\beta_j = p_{c_j} / \sum_{i=1}^{m} q_{ij}^{(k-1)}$ *and rescale each element of this column*

$$q_{ij}^{(k)} = \beta_j q_{ij}^{(k-1)}, 1 \le i \le m.$$

(**Step 5**) *If the convergence criterion*

$$\left| \frac{p_{r_i} - \sum_{j=1}^{n} q_{ij}^{(k)}}{p_{r_i}} \right| < \epsilon, 1 \le i \le m,$$

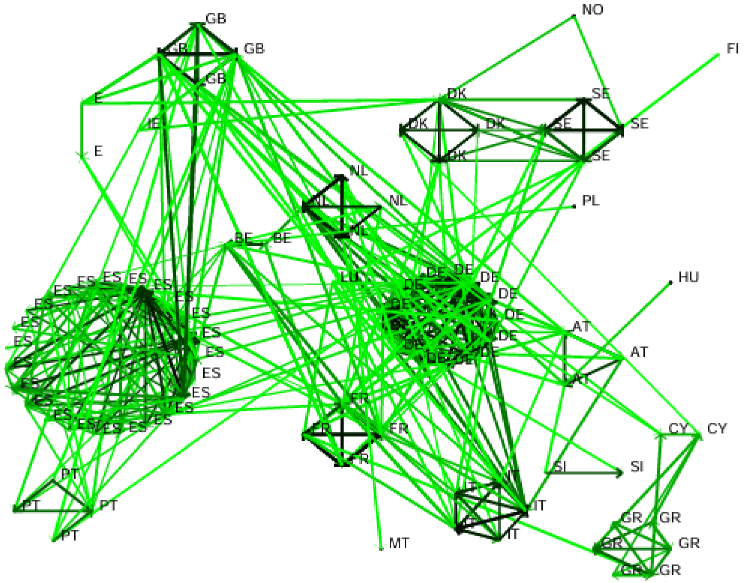
is met then go to ***Step 6****, else go to* ***Step 2****.*

(**Step 6**) $\mathbf{P} = \mathbf{Q}^{(k)}$.

*) Chlebus & Kasapu, 2023, Comptes Rendus Math

How to reconstruct the interbank structure of the EU market?

- **Assumption:** aggregated exposure of the bank and the estimate of the probability that two given banks are in a relationship on the interbank market, where domestic and globally active more likely to interact
- **Question:** how can this structure look like in this system
- **Solution: stochastic block algorithm of** (Hałaj & Kok, 2013)
- **Relavance:** a practical tool (algorithm) for market researchers and financial entities (including supervisors) who have limited access to supervisory data; wide application



An example network of interbank deposits, reconstructed based on balance sheet data and assuming that banks with an international reach and banks active in the same country are more likely to enter into a relationship on the interbank market. Source: Hałaj & Kok (2013)

Simulated networks of Halaj & Kok (2013)

At the initial step 0, the matrix L^0 has all entries equal to 0 and the unmatched interbank assets and liabilities are initiated as $A^0 := A$ and $L^0 := L$. At a step $k + 1$ a pair of banks (i, j) is drawn at random, where all pairs have an equal probability of being selected.

Next, a random number f is drawn from the unit interval that indicates the percentage of bank i's liabilities serviced by bank j. The exposure is updated as follows:

$$L_{ii}^{k+1} = L_{ii}^{k} + f^{k+1} \min\{L_i^k, A_i^k\}$$

and the unmatche $L_i^{k+1} = L_i^k - \sum_{j=1}^{N} X_{ij}^{k+1}$ and $A_j^{k+1} = A_j^k - \sum_{i=1}^{N} X_{ij}^{k+1}$

The stock of interbank liabilities and assets reduces as the volume of the assigned (matched) placements increases. The procedure is repeated until no more interbank liabilities are left to be assigned as placements from one bank to another.

Gandy and Veraart (2016)*

Sample adjacency matrices A from a distribution that generates matrices with given marginals, and assuming that $A_{ij} \sim Bernoulli(prob\ of\ edge\ i \rightarrow j)$, and $L_{ij} | \{A_{ij} = 1\} \sim Exp\left(lambda_{ij}\right)$ determining weights (exposures)

Key idea based on Gibbs sampler: update one or several components of the entire parameter vector by sampling them from their joint conditional distribution given the remainder of the parameter vector. By repeating these updating steps one constructs a Markov Chain whose distribution converges to the target distribution. Here parameter vector is matrix L:

Initialise a chain with matrix L that satisfies r(L) = l, c(L) = a.

MCMC sampler produce a sequence of matrices L1, L2,… and…

…the quantity of interest is: $\boldsymbol{E}[h(L)|a] \cong N^{-1} \sum_{i=1}^{N} h(L^{i\delta+b})$, with

N number of samples, *b* burn-in period, *N* thinning parameter δ

*) Gandy, Veraart (2016), Mang. Science

Gandy and Veraart (2016) in simple terms

Initial matrix (“a guess”)

	50	60	35	35
50	0	40	10	0
55	20	0	5	30
15	10	0	0	5
60	20	20	20	0

Total liabilities

Total assets

1st realization

	50	60	35	35
50	0	42	8	0
55	20	0	5	30
15	10	0	0	5
60	20	18	22	0

2nd realization

	50	60	35	35
50	0	42	8	0
55	20	0	5	30
15	5	5	0	5
60	25	13	22	0

n^th realization

…

0	2	-2	
	0		
		0	
	-2	2	0

Sample a 2x2 block and then sample* shocks that sum(shocks)=0…

0			
	0		
-5	5	0	
5	-5		0

…and repeat with another 2x2 block and assign shocks that sum(shocks)=0

*) sampling from a specific exponential distribution (R package developed by A. Gandy)

Cimini, Garlaschelli,… (2014)

Configuration models (for directed networks): find a distribution such that if we sample from it, we get a network with desired properties (e.g., degree distribution)

The maximum likelihood method requires to solve the following system of 2N equations[1]:

$$InDegree(i) = \sum_{j \neq i} \frac{x_i y_j}{1 + x_i y_j}$$

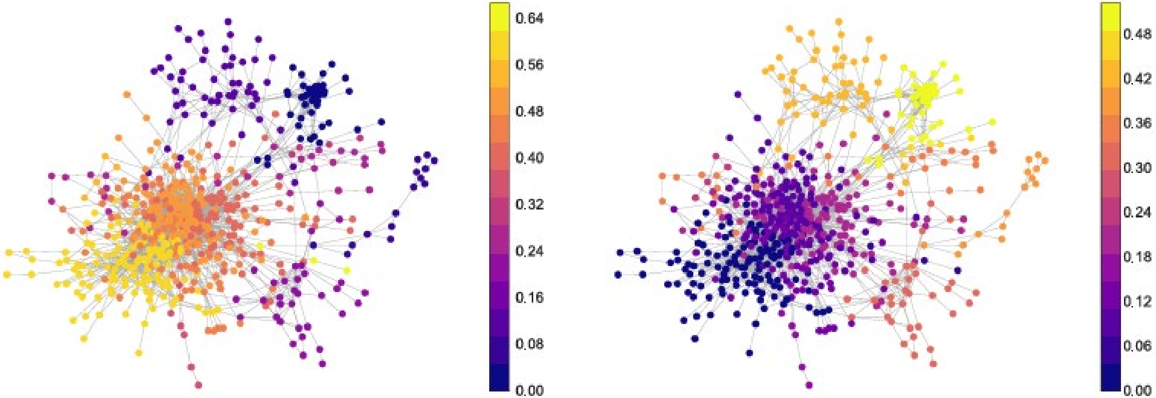
$$OutDegree(i) = \sum_{j \neq i} \frac{x_j y_i}{1 + x_j y_i}$$

and $\frac{x_i^* y_j^*}{1+x_i^* y_j^*}$ is a probability of a link between i and j for x* and y* solving the equations

[1]) derived from a specific max likelihood function

Example: alternative world trade networks that preserve the properties of connectivity…

…i.e., countries similar in terms of trading similar products trade those product but perhaps with different countries



Projection of the WTW in the year 2000, with colors indicating the intensity of trade activity of ‘advanced’ economies (left panel) and ‘developing’ economies (right panel) over the products communities.

Source: Sarecco et al. (2017) New J Phys

A twist: configuration models not only to generate networks but to compare structures

On 9 March 2012, the International Swaps and Derivatives Association (ISDA) announced that the triggering of the collective action clauses in the domestic-law bonds represented a “Restructuring” credit event for the CDS contracts on the Hellenic Republic. **What was the impact of the Greek credit event on the CDS market?**

The estimates of the mean in- and out-degrees are:

$$\bar{I}_i = \sum_{j \neq i} p^G_{j,i} \text{ and } \bar{O}_i = \sum_{j \neq i} p^G_{i,j}$$

And standard deviation:

$$\sigma^G = \left(\sum_{i,j} p^G_{i,j} (1 - p^G_{i,j}) \right)^{1/2}$$

→

$$z = \frac{\left(\| I - \bar{I} \|^2 + \| O - \bar{O} \|^2 \right)^{1/2}}{\sigma^G}$$

Consequence of the default: market of CDS exposures for Greece and for EU periphery changes…

…with potentially negative impact on risk sharing

	201103	201204	201212	201410
Net long exposures				
Greece	3.2	6.9	6.4	6.3
Bank-periphery	4.0	4.8	5.1	5.7
Dealers	4.6	4.9	5.6	6.6
Emerging markets	4.0	4.5	4.8	5.3
EU periphery sov	4.1	5.4	5.7	6.3

In the table: Z-scores

We observe material increase of the score, indicating significant structural change of the network for Greece and, relatedly, EU periphery sovereign names

Digression: what is the uniform distribution of matrices* with entries satisfying some marginals?

- Consider the following function (entropy-like terms are not a coincidence), on the elements of a matrix $0 \leq x_{ij} \leq 1$ (why is the domain well-defined?)

$$h(X) = \sum_{i,j} x_{ij} \ln\left(\frac{1}{x_{ij}}\right) + (1 - x_{ij})\ln(\frac{1}{1 - x_{ij}})$$

*) Based on Barvinok (2009) “What does a random contingency table look like?”, showing how interdisciplinary the topics is and how deeply mathematical the foundations of the systemic risk modelling are

Characterization for adjacency matrices…

- Suppose that polytope* P0(R,C) has a non-empty interior and let Z0 ∈ P0(R,C) be the maximum entropy matrix. Let X = (x_{ij}) be a random *m*×*n* matrix of independent Bernoulli random variables x_{ij} such that $\boldsymbol{E}X$ = Z0. Then:

- **The probability mass function of X is constant on the set A0(R,C) of “0-1” matrices with row sums R and column sums C and**

$$\mathbf{Pr}\{\boldsymbol{X} = \boldsymbol{D}\} = \boldsymbol{e}^{-\boldsymbol{h}(\boldsymbol{Z}_0)}$$

*) polytope – another way to say a set of matrices with given sum of rows and columns, additionally requiring that entries are between 0 and 1.

…and for weighted adjacency matrices

- Theorem from the previous slide implies that in many respects a random matrix D ∈ A0(R,C) behaves as a random matrix X of **independent Bernoulli random variables** such that $\boldsymbol{E}X$=Z0, where Z0 is the maximum entropy matrix.

But we can generalize further and:

- The theorem (with some additional but not excessively stringent assumptions), implies that in many respects a random matrix D ∈ A+(R,C) behaves as a matrix X of **independent geometric random variables** such that $\boldsymbol{E}X$ = Z+, where Z+ is the maximum entropy matrix.

Systemic Risk Measures

How to measure systemic risk based on the structure of the network representing connectivity?

- Networks are just one approach among other methods
  - With market, high-frequency data, impact of a bank on the system and vice versa can be estimated: SRISK, CoVaR, etc.
- Even if not explicitly using networks, they are a natural outcome
  - Simple VAR (or Halaj and Hipp (2024) for a more discussion)
- Systemic risk not all about interconnectedness
  - Other characteristics, jointly with interconnectedness, create vulnerabilities, high leverage or poor risk management and governance

Structural measures – information about the configuration of nodes and links

- Sparsity, fragmentation, layers, (useful for larger networks, with multiple dimensions)
  - Sparsity: low number of connections compared to the number of nodes (banks)
  - Fragmentation: blocks of nodes, with nodes within block well connected and blocks loosely connected
  - Layers: links in distinctive markets (equity or derivatives)
- Centrality, i.e., importance of single node (bank) for the system
  - Based on links and sequence of links (paths), i.e., how much they link different parts of the system

Centrality measures – information about the nodes and their connectivity with the rest of the system

- Degree centrality (fraction of nodes it is connected to)
  - Direct connectedness to identify banks/ firms well connected to the system, with interpretation depending on the context
    - \+ More diversified client base/ funding sources help to increase resilience to the shocks
    - \- Higher risk of spreading shocks or receiving shocks
- Very simple measure and, despite limitations, a great first stab at the contagion risk identification

Centrality measures – information about connections spread across the system

- Eigenvector centrality (computes the centrality for a node by adding the centrality of its predecessors)
  - Takes the structure of the whole system to identify nodes (banks) that are connected to banks that are well connected to other banks that are connected to…
    - \+ "Well diversified funding providers are less likely to experience problems and pass them onto me"
    - \- "If I am hit hard, I am more likely to adversely impact not only my direct creditors but also other institution relying on interbank funding"

Centrality measures – a step towards more general methods

- Katz centrality (centrality for a node based on the centrality of its neighbors)
  - Mathematically, centrality x is a solution to the following implicit equation:

$$x_i = \alpha \sum_j A_{ij} x_j + \beta,$$

with A as adjacency matrix (A_{ij}=1 in case of a link, 0 otherwise), extra weight β to immediate neighbours, and distant neighbours penalized by α.

+ “Well diversified funding providers are less likely to experience problems and pass them onto me”

- “If I am hit hard, I am more likely to adversely impact not only my direct creditors but also other institution relying on interbank funding”

Centrality measures – more tailored to financial system considering banks leverage

- Debt rank (take Katz centrality but what if weights β depend on the leverage of the institutions)
  - $x= \beta *v + \alpha* \beta * x$, with $\beta_{ij} := \min\{1, exposure_{ij}/capital_j\}$…
  - And economic value of the impact of i on j by multiplying the impact by the relative economic value of the node j, $v_j = exposure_j/\Sigma_j exposure_l$.
    - “My financial conditions may deteriorate when other default or just are in distress (indirect channels of contagion, sentiment-based)
    - No default needed for contagion propagation; a bank goes in distress when a predecessor just went in distress and so recursively

Centrality measures – Merton meets "Mr and Mrs Contagion"

- Network Valuation indicator* (take debt rank but try to endogenize the value of banks equity based on the fair value of exposures, <u>including fair value of exposures in the network</u>)
  - Indicator is defined by a stressed capital
  - c^S_i = assets_i – liabilities_i + $\sum_{j=1 \ldots N} exposure_{ij}$ *Value(c^S) – $\sum \; ibank\_liabilities$
    - "Value of my capital, even if not seen in the financial statements as yet, drops if my debtors are in distress, i.e., value of their capital drops…"
    - "…or other banks do not pay their interbank obligations, not only to me but to my debtors"

**Python code and example: https://github.com/marcobardoscia/neva*

Centrality measures – market-based approach

- CoVaR (centrality based on the market data)
  - Mathematically, centrality x is a solution to the following implicit equation:

$$\boldsymbol{P}\left(loss_i \leq \boldsymbol{CoVaR}(q, i, j)\,\middle|\, loss_j \leq VaR(q, i)\right) = q$$

with *loss* being a (negative) return on a given name's equity, VaR is Value-at-Risk at with quantile q.

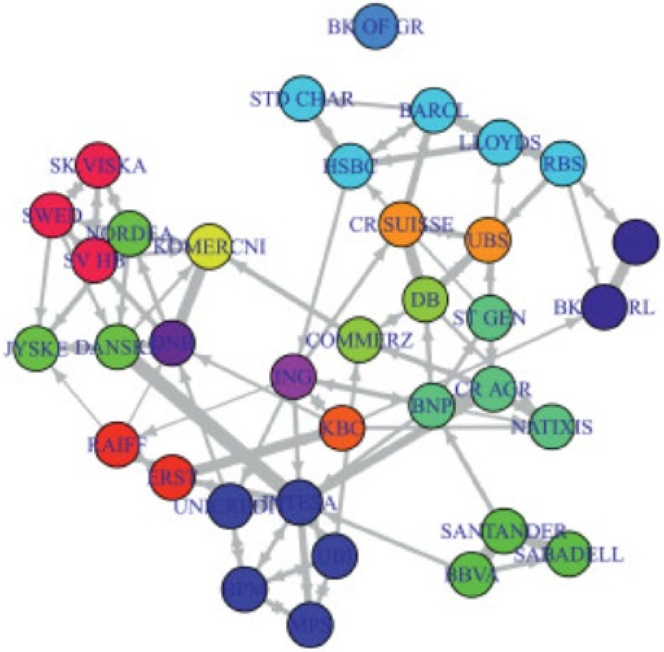
+ Data availability, as one can use public info on the time-series of returns

- Investor-centric view, i.e., perhaps not straightforward to interpret the output for the financial stability, pair-wise view, i.e., what about multiple entities in distress* and historical correlations between variables (returns) assumed to remain valid for the future (risk is about the future)

*) Marginal Expected Shortfall partly addresses this concern

Case study – European banks' tail connectedness*

- Delta CoVaR on equity (stock) returns
- 36 Banks included in the EBA stress test
- CC-GARCH model (Engle, 2002) applied to estimate the conditional quantile model on the standardized innovations, 2007-2012
- Result: "Institutions from the same country (highlighted with the same color) tend to connect more tightly to each other compared to the other banks in the system."



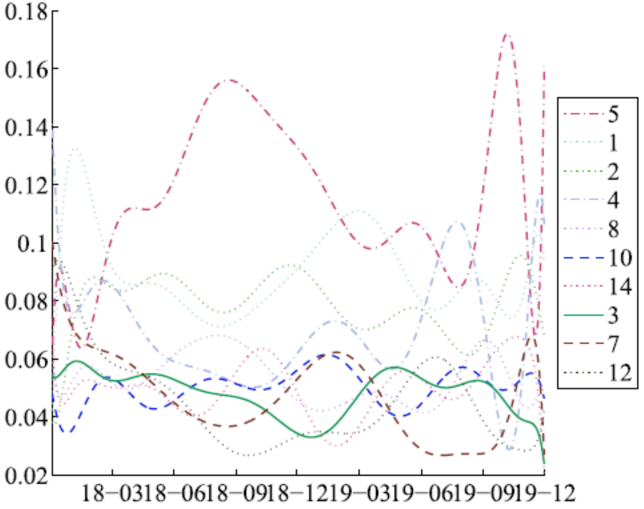
*) Torri et al, 2021, JEDC

Case study – Peru: Monitoring tools*

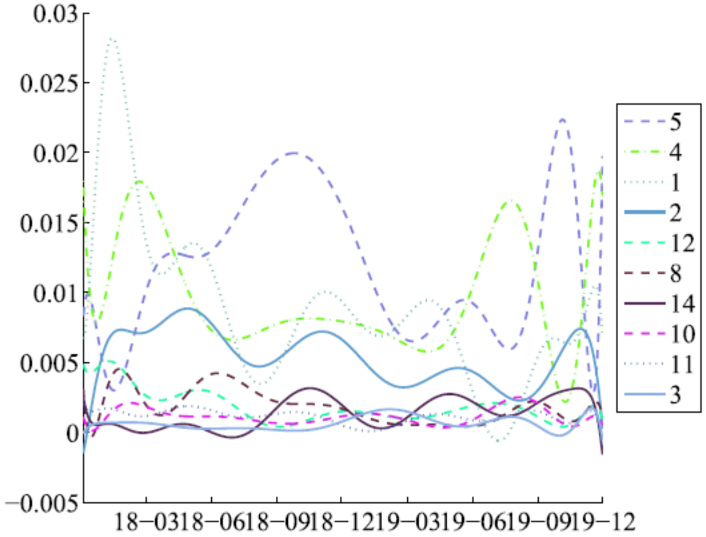
- Multi-layer network of exposures based on reporting requirements set by the supervisory authority
- 56 FIs – commercial banks, investment banks, development banks, microfinance institutions
- Daily and monthly frequency, depending on the layer:
  - Daily: short-term IB borrowing
  - Monthly: long-term credit, demand and term deposits, security cross-holdings, and derivatives
- There are data gaps but this model is an important initiative to demonstrate that even partial data can allow for setting up a systemic risk monitoring framework

*) Cuba et al, 2021, Lat. J. Cent. Banking

Case study – Peru: Centrality and DebtRank



(a) 10 highest centrality.

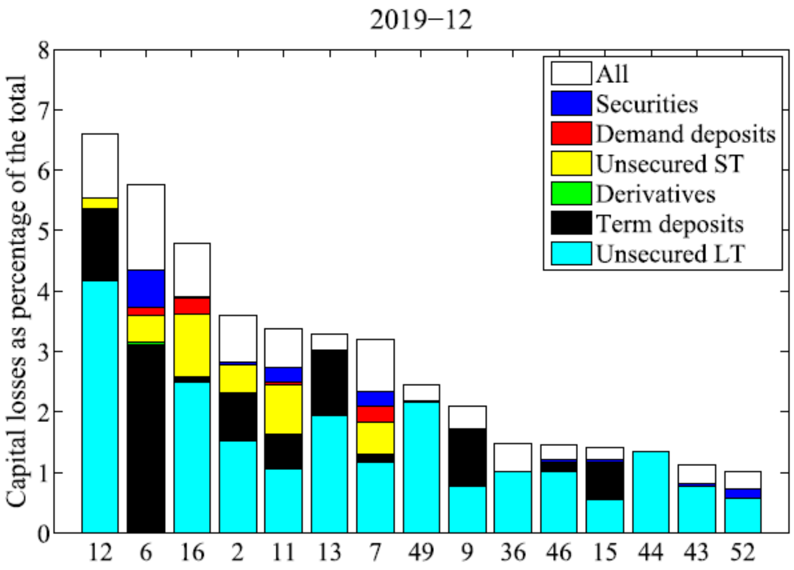


(a) 10 highest DebtRank

“Other measures, like completeness and clustering not very informative, with overlapping degree as a potentially useful one (Using the stochastic block model, [the authors] were able to identify two communities of banks, corresponding to institutions with relatively high/low degrees of overlapping.”

Case study – Peru: attribution analysis

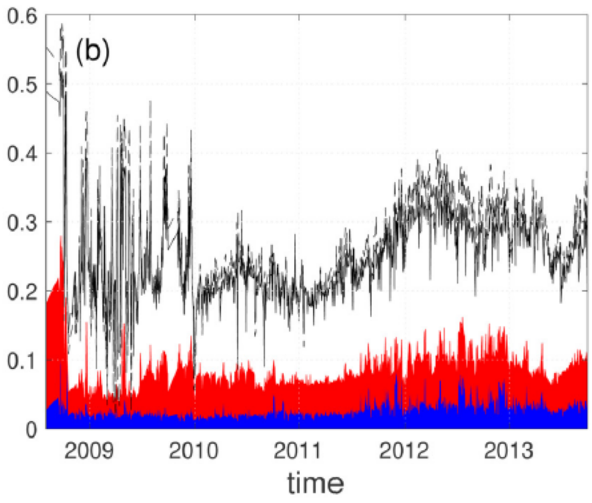
- What drives losses, i.e., which layer of exposures?
- Top 15 institutions with the highest DebtRank and the contribution of each layer.
- Dashboards help to monitor risks, depicting outliers (in terms of concentration, specific exposures to potentially vulnerable sectors)



Top 15 institutions concerning the losses indicated by DebtRank, with a breakdown by market (layer of the network).

Case study – portfolio overlaps: how much data is good enough*

- Trade-off:
  - More data collected and granular, detailed projections
  - Less effort to collect and clean data but less detailed insights
- Authorities must make a conscious and informed decision:
  - Materiality of risks: bang for the buck?
  - Robust measures of instability: could conclusions change if more granular portfolio/ product breakdowns are considered?
  - Mexico: e.g., daily transactions collected on interbank exposures and derivative transactions



DebtRank using a multilayer network of exposures, based on data collected (even) on daily basis.
Red= contribution of overlapping portfolios
Blue= impact from direct exposures

*) Poledna et al, 2015, JFS

Case study - Global: not a one-fit-all approach*

Authors	Category	Description
Anand et al. (2015)	Sampling	Minimises the number of links necessary for distributing a given volume of loans
Baral and Fique (2012)	Iterative	Exposure sizes drawn from a copula fitted to the aggregate exposures of all banks
Cimini et al. (2015)	Sampling	A fitness model determines the likelihood of directed linkages and exposures
Drehmann and Tarashev (2013)	Iterative	Postulates that the network should have a core of banks with large exposures between themselves, and a periphery of other banks with smaller exposures
Halaj and Kok (2013)	Sampling	Links are drawn at random, where all links have an equal probability, and exposures are assigned according to an iterative procedure
Upper and Worms (2004)	Iterative	The standard maximum-entropy method
Musmeci et al. (2013)	Sampling	A fitness model determines the likelihood of undirected linkages, and exposures are allocated via *Maxe*

*) Anand et al, 2018, JFS

Case study - Global: network statistics used as yardsticks in the horse race

	BIS1	BR01	CA01	DE01	DK01	EU01	HU01	IT01	KR01	MX01	MX03	MX06	NL01
Number of nodes	31	111.9	6	592.4	14	26	35.8	535.4	18	43	43	43	159
Number of links	742.7	512.7	29.5	11623.5	77	197.7	274.8	3158.9	263	408.3	127.3	53.3	546.1
Density	79.9	4.1	98.3	3.3	42.3	29.2	22.2	1.1	85.9	22.6	7.1	3	2.2
Average degree	24	4.6	4.9	19.6	5.5	7.6	7.7	5.9	14.6	9.5	3	1.2	3.4
Median degree	25.3	2.2	5	14.7	5	8.4	7.8	3.2	15	9.3	2	1	1
Assortativity	−.19	−.37		−.6	−.3	−.33	−.31	−.43	−.17	−.22	−.23	−.39	−.49
Clustering	28.5	4.4	67.3	40.3	21.5	15.5	22.2	19.1	21.2	12.9	5.9	4.3	6.6
Lender dependency	28.7	65.2	35.8	43.6	37.5	71.2	32.4	71.6	31.6	54.6	71.7	84.6	78.6
Borrower dependency	30.6	59.8	40.2	69.4	39.6	71.1	39.4	87.8	24.9	51.8	61.6	74.8	76.2
Mean HHI assets	.16	.5	.26	.3	.27	.46	.24	.64	.19	.39	.47	.54	.54
Median HHI assets	.15	.44	.25	.22	.25	.35	.14	.61	.17	.33	.42	.6	.57
Mean HHI liabilities	.16	.38	.29	.59	.25	.56	.25	.84	.15	.36	.33	.24	.48
Median HHI liabilities	.13	.31	.26	.59	.23	.6	.11	1	.14	.26	.25	0	.45
Core size (% banks)	73.1	9.8	76.7	6.6	42.9	36.3	31.5	3.5	77.8	31	16.3	7	6.5
Error score (% links)	3.5	41.3	1.4	12.5	14.3	12.2	22.1	22.8	3.4	24.7	39.4	55.2	25.2
Number of slices	3	12	10	12	1	9	12	10	1	3	3	3	10

What have we learned?

- **We can create a meaningful network representation of the system, to study its features based on network characteristics, even if we do not have full information about connections between the nodes (e.g., exposures between financial institutions)**
- **Network measures offer a toolkit to glean from on the properties of the system that may help to indicate pockets of systemic risk**